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Physics Procedia 27 (2012) 84 – 87

Physics Procedia

ISS2011
Spin Hall effect in a superconductor/normal metal junction

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Abstract

We theoretically study a spin Hall effect of normal metal in a superconductor/normal metal (SN) junction based on perturbation theory. The skew scattering and side jump contributions are both taken into account to calculate the spin Hall conductivity (SHC) in the normal metal. We fin that both contributions are anomalously enhanced when the voltage (eV) between the superconductor and the normal metal approaches to the superconducting gap (Δ_0). We also fin that the SHC at $eV = \Delta_0$ increases with decreasing the junction resistance (R_{T}). These results clearly demonstrate that the SHC in SN junctions can be controlled by applying an external dc electric fiel and by varying R_{T}, suggesting that SN junctions have potential applications for spintronic devices with large spin Hall effects.

Keywords: spin Hall effect, spintronics, superconductor, normal metal, tunnel junction, nanostructure

1. Introduction

Instead of the charge current as in conventional electronics, spintronics utilizes the spin current, which is the fl w of spin angular momentum of electron [1]. For application purposes of spintronic devices, the technique to control the spin current by an external electric fiel is essential because the electric fiel can control the fl w of electrons in nanometer scale devices. In this regard, an interaction between the spin and orbital motion of electrons [spin-orbit interaction (SOI)] is an important ingredient. The SOI induces the novel phenomenon called spin Hall effect (SHE), where a charge current induces spin dependent motion of electrons, fl wing perpendicular to the charge current and in the opposite directions for up- and down-spin electrons. The SHE has been recognized as a key effect to convert the charge current into the spin current and vice versa [2, 3, 4].

The SHE was firs predicted theoretically decades ago, and now it is well accepted that there are two types of SHE, the one caused by the SOI of a host metal (*intrinsic* SHE) [2] and the other caused by the SOI of nonmagnetic guest impurities in a host metal (*extrinsic* SHE) [3]. For the extrinsic SHE, there are two contributions, skew scattering and side jump. The skew scattering results from the impurity scattering

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 doi:10.1016/j.phpro.2012.03.416

via the SOI [5], whereas the side jump is due to the anomalous velocity induced by the SOI [6]. The firs experimental observation of the extrinsic SHE has been reported by Kato *et al*, who have detected the spin accumulation induced by the extrinsic SHE in GaAs systems [7].

One of the important current issues is to fin a way to obtain a large SHE [8] A large spin Hall conductivity (SHC) has an ability to generate the large spin current because the spin current induced by the SHE is proportional to the SHC [9]. The SHC in turn depends sensitively on the SOI as well as the impurity scattering in a host material. The SHE observed so far is, however, still small, thus requiring sensitive experimental measurements to detect the effect [7, 10]. Therefore, findin an alternative way to further increase the SHE is highly desirable, which certainly helps to achieve a variety of SHE based spintronic devices in the future.

In this paper, we theoretically study a SHE in a superconductor/normal metal (SN) junction, based on perturbation theory. Taking into account both contributions of the skew scattering and side jump contributions in low impurity concentrations, we show that the extrinsic SHC in the normal metal is anomalously enhanced when the voltage (eV) between the superconductor (S) and the normal metal (N) approaches the superconducting gap (Δ_0). We also fin that the SHC at $eV = \Delta_0$ increases with decreasing the junction resistance (R_{T}). Our results demonstrate a way to control and amplify the SHC by applying an external dc electric fiel and by varying R_{T}, suggesting that SN junctions have potential applications for spintronic devices with large SHEs.

2. Formulation of spin Hall conductivity in SN junction

The system considered is an s-wave SN junction as depicted in Fig. 1. An insulating barrier is inserted between S and N to suppress the proximity effect. In this setup, the thickness of the N is considered thin enough to treat the N as a two dimensional N. A dc bias voltage V_{bias} is applied in the x-direction to fl w electrons in the N. The chemical potential difference between S and N is adjusted by a dc voltage V applied in the z-direction [11]. The system is thus described by the Hamiltonian $H = H_{\mathrm{S}} + H^{\mathrm{N}}_{\mathrm{em}} + H_{\mathrm{N}} + H_{\mathrm{T}}$. Here H_{S} is the BCS Hamiltonian with an s-wave superconducting gap. H^{N}_{em} represents the interaction with the applied dc bias voltage V_{bias}: $H^{\mathrm{N}}_{\mathrm{em}} = -\int d^2r\boldsymbol{j}(\boldsymbol{r},t)\cdot\boldsymbol{A}(t)$ where $\boldsymbol{j}(\boldsymbol{r},t)$ and $\boldsymbol{A}(t)$ are a current operator (define below) and a vector potential, respectively. The gauge is set to satisfy $\boldsymbol{E} = -\partial_t\boldsymbol{A}(t)$ with a spatially uniform electric fiel [$\boldsymbol{A}(t) = (-E_x t, 0, 0)$]. The N is described by $H_{\mathrm{N}} = \sum_\sigma \int d^2r c^\dagger_\sigma(\boldsymbol{r})\left(-\frac{\hbar^2}{2m}\nabla^2 - \mu_{\mathrm{F}}\right)c_\sigma(\boldsymbol{r}) + H_{\mathrm{imp}} + H_{\mathrm{SOI}}$, where $c_\sigma(\boldsymbol{r})$ is an annihilation operator of electron with spin σ at position $\boldsymbol{r}$. m and μ_{F} are mass of electron and the Fermi level, respectively. The terms H_{imp} and H_{SOI} describe a nonmagnetic impurity scattering and the SOI, respectively,

$$\begin{aligned} H_{\mathrm{imp}} &= \sum_\sigma \int d^2r u(\boldsymbol{r}) c^\dagger_\sigma(\boldsymbol{r}) c_\sigma(\boldsymbol{r}), \\ H_{\mathrm{SOI}} &= -i\lambda_{\mathrm{SO}} \sum_{\alpha,\beta} \int d^2r c^\dagger_\alpha(\boldsymbol{r}) \left[\nabla u(\boldsymbol{r}) \cdot \nabla \times \boldsymbol{\sigma}_{\alpha,\beta}\right] c_\beta(\boldsymbol{r}), \end{aligned}$$

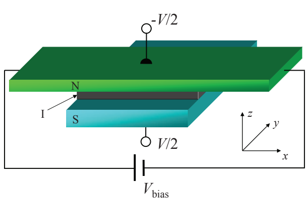

Fig. 1. Schematic configuratio of a SN junction studied to induce a large SHE. V_{bias} is an applied dc voltage at the opposite edges of the N. V is a voltage applied between S and N. An insulating barrier (I) is inserted between S and N.

where $u(\boldsymbol{r}) = u_0 \sum_i \delta(\boldsymbol{r} - \boldsymbol{R}_i)$ is an impurity potential with the strength u_0 locating at $\boldsymbol{R}_i$ in the N. $\lambda_{\rm SO}$ is the SOI coupling and $\boldsymbol{\sigma}_{\alpha,\beta}$ are the Pauli matrices. For the tunneling of electrons between S and N, we adopt the tunneling Hamiltonian $H_{\rm T}$ described by

$$H_{\rm T} = \sum_{\sigma} \int_{\boldsymbol{r}\in{\rm N},\boldsymbol{r}'\in{\rm S}} d^2 r d^3 r' T_{\boldsymbol{r},\boldsymbol{r}'} e^{i\frac{eV}{\hbar}t} c^{\dagger}_{\sigma}(\boldsymbol{r}) d_{\sigma}(\boldsymbol{r}') + \text{h.c.}.$$

Here, $d_\sigma(\boldsymbol{r})$ is an annihilation operator of electron in the S and the tunneling matrix element $T_{\boldsymbol{r},\boldsymbol{r}'}$ is non zero only at the SN boundary, i.e., $T_{\boldsymbol{r},\boldsymbol{r}'} = T_0 \delta(\boldsymbol{r} - \boldsymbol{r}'_{\parallel})\delta(z')$ with $\boldsymbol{r}'_{\parallel} = (x', y', 0)$. Finally, the voltage V between S and N is described by the exponential factor $e^{ieVt/\hbar}$ in $H_{\rm T}$.

To evaluate the extrinsic SHC within the linear response theory, firs we consider the statistical average of the following two current operators in the y-direction, $j^{\rm N}_{y,\sigma}(\boldsymbol{r}, t) = -i\frac{e\hbar}{mS_A} \sum_{\boldsymbol{k},\boldsymbol{q}} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} k_y \left\langle G^{-+}_{\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\sigma,\sigma}(t, t)\right\rangle_{\rm i}$ and $j^{\rm SO}_{y,\sigma}(\boldsymbol{r}, t) = i\frac{e\lambda_{\rm SO}}{\hbar S_A} \sum_{\boldsymbol{k},\boldsymbol{q},\sigma'} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \left\langle [\nabla U(\boldsymbol{r}) \times \boldsymbol{\sigma}_{\sigma'\sigma}]_y \, G^{-+}_{\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\sigma,\sigma'}(t, t)\right\rangle_{\rm i}$ [12], where $G^{-+}_{\boldsymbol{k}',\boldsymbol{k},\sigma',\sigma}(t', t) = i\left\langle c^{\dagger}_{\boldsymbol{k},\sigma}(t) c_{\boldsymbol{k}',\sigma'}(t')\right\rangle$ is a lesser Green's function, $\boldsymbol{k}$ and $\boldsymbol{k}'$ are wave numbers of electrons, S_A is the area of the junction, and $\langle\cdots\rangle_{\rm i}$ represents the impurity average. The lesser Green's function is derived from the contour Green's function, $G_{\boldsymbol{k},\boldsymbol{k}',\sigma,\sigma'}(t, t') = -i\left\langle T_c c_{\boldsymbol{k},\sigma}(t) c^{\dagger}_{\boldsymbol{k}',\sigma'}(t')\right\rangle$, where $\langle\cdots\rangle$ denotes the quantum statistical average at zero temperature and T_c is a contour ordering operator. $j^{\rm N}_{y,\sigma}(\boldsymbol{r}, t)$ $[j^{\rm SO}_{y,\sigma}(\boldsymbol{r}, t)]$ is the normal (anomalous) current, from which the skew scattering (side jump) contribution is obtained. The SHC is obtained from $j^{\rm N(SO)}_{y,\sigma}(\boldsymbol{r}, t) = \sigma\sigma^{\rm SS(SJ)}_{xy} E_x$, where $\sigma^{\rm SS(SJ)}_{xy}$ is the skew scattering (side jump) contribution to the SHC, and $\sigma = +1$ (-1) for up (down) electrons. The terms $H_{\rm imp}$, $H_{\rm SOI}$, and $H_{\rm T}$ are treated within a perturbation theory keeping the lowest order contributions of skew scattering and side jump in the SN junction. This approximation is valid in a low impurity concentration $n_{\rm imp}$ and in a tunnel junction limit with small T_0. The SHC due to the skew scattering and side jump contributions in the SN junction is then given respectively as

$$\frac{\sigma^{\rm SS}_{xy}}{\tilde{\sigma}^{\rm SS}_{xy}} = 1 + \frac{4T_0^2}{\hbar^3\tau} \int_{-\infty}^{\infty} d\xi \mid g^{\rm R}_{\boldsymbol{k}} \mid^2 {\rm Re}\left[g^{\rm R}_{\boldsymbol{k}} g^{\rm R}_{{\rm S},\boldsymbol{k}}(eV)\right] - \frac{2T_0^2}{\hbar^3} \int_{-\infty}^{\infty} d\xi {\rm Im}\left[\left(g^{\rm R}_{\boldsymbol{k}}\right)^2 g^{\rm R}_{{\rm S},\boldsymbol{k}}(eV)\right],$$

$$\frac{\sigma^{\rm SJ}_{xy}}{\tilde{\sigma}^{\rm SJ}_{xy}} = 1 + \frac{T_0^2}{\hbar^3\pi\tau} \int_{-\infty}^{\infty} d\xi \mid g^{\rm R}_{\boldsymbol{k}} \mid^2 {\rm Re}\left[g^{\rm R}_{\boldsymbol{k}} g^{\rm R}_{{\rm S},\boldsymbol{k}}(eV)\right] - \frac{T_0^2}{\hbar^3\pi} \int_{-\infty}^{\infty} d\xi {\rm Im}\left[\left(g^{\rm R}_{\boldsymbol{k}}\right)^2 g^{\rm R}_{{\rm S},\boldsymbol{k}}(eV)\right],$$

where $\tilde{\sigma}^{\rm SS(SJ)}_{xy}$ is the skew scattering (side jump) contribution to the SHC in the bulk N with $T_0 = 0$. $g^{\rm R(A)}_{\boldsymbol{k}} = \hbar/(-\xi \pm i\hbar/2\tau)$ is the retarded (advanced) Green's function in the N. $\xi = \hbar^2 k^2/2m - \mu_{\rm F}$ is the kinetic energy of electron and τ is the relaxation time due to the nonmagnetic impurity scattering within the Born approximation. $g^{\rm R(A)}_{{\rm S},\boldsymbol{k}}(eV) = -i\frac{m}{2\hbar}\left[\frac{eV}{i\Omega_{\rm R(A)}}\left(\frac{1}{p_\uparrow} + \frac{1}{p_\downarrow}\right) + \frac{1}{p_\uparrow} - \frac{1}{p_\downarrow}\right]$ is the diagonal part of the retarded (advanced) Green's function in the S which satisfie the Gorkov's equation. Here, $p_{\uparrow(\downarrow)} = \sqrt{2m(-\xi \pm i\Omega_{\rm R(A)})/\hbar^2}$ and $i\Omega_{\rm R(A)} = \sqrt{(eV \pm i\eta)^2 - \Delta_0^2}$. η is the inelastic scattering rate in the S [13] and Δ_0 is the superconducting gap at zero temperature.

3. Numerical results

Let us now evaluate numerically the SHC for the two contributions derived above. As an example, here we take $\eta/\Delta_0 = 1 \times 10^{-3}$ [14]. Fig. 2 (a) shows the skew scattering contribution to the SHC ($\sigma^{\rm SS}_{xy}$), normalized by the SHC for the bulk N ($\tilde{\sigma}^{\rm SS}_{xy}$), as functions of V and the inverse relaxation time τ^{-1}. It is observed in Fig. 2 (a) that $\sigma^{\rm SS}_{xy}$ is almost the same as that of the bulk system when eV deviates from Δ_0. However, when eV approaches to Δ_0, $\sigma^{\rm SS}_{xy}$ becomes anomalously enhanced. Moreover, it is seen that $\sigma^{\rm SS}_{xy}$ monotonically increases with increasing τ [16]. Fig. 2 (b) shows the side jump contribution to the SHC ($\sigma^{\rm SJ}_{xy}$), which exhibits the similar characteristic behavior, i.e., large enhancement of $\sigma^{\rm SJ}_{xy}$ for eV close to Δ_0, although the enhancement factor for $\sigma^{\rm SJ}_{xy}$ appears smaller than that for $\sigma^{\rm SS}_{xy}$. These results clearly demonstrate that $\sigma^{\rm SS}_{xy}$ and $\sigma^{\rm SJ}_{xy}$ can be significantl amplifie by tunning V between S and N in the SN junction.

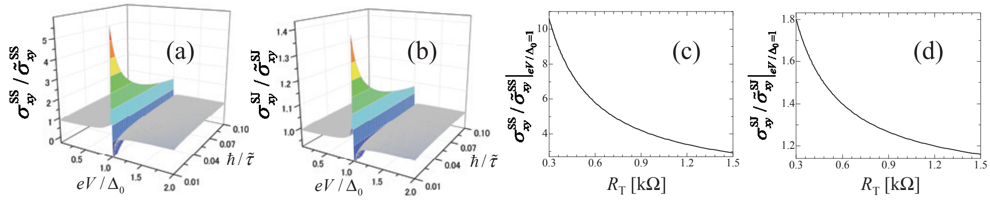

Fig. 2. The SHC for the skew scattering (a) and side jump (b) contributions as functions of the voltage V and the inverse relaxation time τ^{-1}. Here $\tilde{\tau} = \tau\Delta_0$, $\tilde{\sigma}_{xy}^{\mathrm{SS}}$ ($\tilde{\sigma}_{xy}^{\mathrm{SJ}}$) is the skew scattering (side jump) contribution in the bulk N, and a dimensionless parameter $g_0 = \frac{T_0^2\sqrt{m}}{\hbar\pi(2\Delta_0)^{3/2}}$ is set to be 1×10^{-5} [15]. The SHC for the skew scattering (c) and side jump (d) contributions as a function of the junction resistance R_{T}. Here $eV = \Delta_0$ and $\hbar/\tilde{\tau} = 10^{-2}$.

Finally, we examine the dependence of the SHC with varying the junction resistance (R_{T}). It is well known that R_{T} for junctions with T_0 is given by $R_{\mathrm{T}} = \hbar/(4\pi e^2 N_{\mathrm{F}} N_{\mathrm{F}}^{\mathrm{S}} S_A^2 T_0^2)$, where N_{F} and $N_{\mathrm{F}}^{\mathrm{S}}$ are the electron density of states (DOS) at the Fermi level in N and S of the normal conducting state, respectively [17]. Therefore, $R_{\mathrm{T}} \approx 6 \times 10^{-3}/g_0$ for $S_{\mathrm{A}} = 0.1\ \mu\mathrm{m} \times 100$ nm and $\mu_{\mathrm{F}} = 1$ eV when the free electron model is adopted [18]. Figure 2 (c) and (d) show the SHC ($\sigma_{xy}^{\mathrm{SS(SJ)}}/\tilde{\sigma}_{xy}^{\mathrm{SS(SJ)}}$) for $eV = \Delta_0$ and $\hbar/\tilde{\tau} = 10^{-2}$ with varying R_{T}. One can clearly see that the SHC for both contributions monotonously increases with decreasing R_{T}. These results suggest that R_{T} is also an important quantity to determine the SHC.

4. Summary

In summary, we have theoretically studied the SHE in a SN junction, based on perturbation theory. The skew scattering and side jump contributions have been taken into account to calculate the SHC in low impurity concentrations. We found that both contributions are anomalously enhanced when eV between S and N is adjusted close to Δ_0. We have also examined R_{T} dependence on the SHC, and found that the SHC at $eV = \Delta_0$ increases with decreasing R_{T}. We believe that the enhancement of the SHC is large enough to be observed experimentally. Our results demonstrate that the SHC can be controlled and amplifie by using the dc electric fiel and by varying R_{T}, suggesting that SN junctions has potential applications for spintronic devices with large SHEs.

A part of the calculation has been performed using RIKEN Integrated Cluster of Clusters (RICC).

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